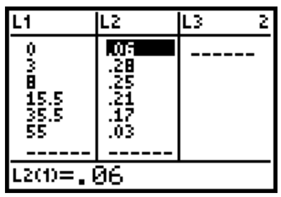
CHAPTER 7 PROBABILITY DISTRIBUTIONS

Section 7.1 ▪ Random Variables and Expected Value

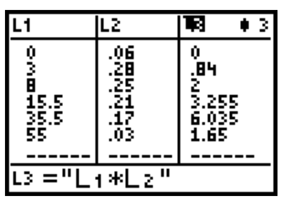
Expected Value and Standard Deviation

You can practice computing the expected value, variance, and standard deviation of a probability distribution table by using the spreadsheet capabilities of the List Editor. For example, consider the data in Display 7.3 and the calculations on pages 381–382 of the *Statistics in Action* student text.

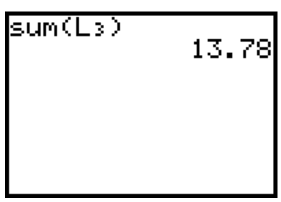
a. Enter the probability distribution table into lists **L1** and **L2**.



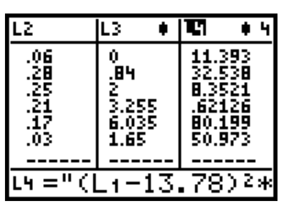
b. Define list **L3** as the products of the values and the probabilities.



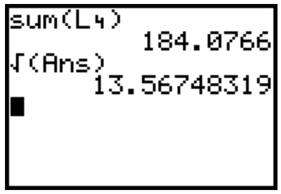
c. The expected value is the sum of list **L3**. On the Home screen, calculate **sum(L3)**. Find the **sum(** command by pressing 2nd **[LIST] MATH 5:sum(**.



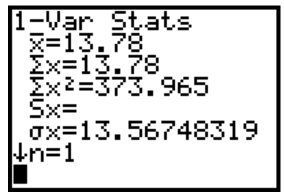
d. Define list **L4** as the products of the squares of the deviations from the mean and the probabilities, $(L1-13.78)^2*L2$.



e. The variance is the sum of list L4. The standard deviation is the square root of the variance.



More directly, the expected value and standard deviation can be calculated using the **1-Var Stats** command. You find this command by pressing STAT **CALC 1:1-Var Stats**, and, for a frequency table, you follow the command with the names of two lists separated by a comma. The first list you enter must contain the values, and the second list must contain the relative frequencies.



Note that the calculator displays no value for **Sx** when list L2 contains relative frequencies. The calculator assumes that you are summarizing a probability distribution for a population and therefore omits the sample standard deviation.

Rules for Means and Variances

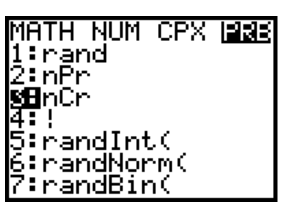
The **1-Var Stats** command makes it relatively easy for you to explore and recall the rules listed on page 385 of the *Statistics in Action* student text. For more information on how to do this, review "Properties of Summary Statistics" in Chapter 2, page 23, of this Calculator Guide.

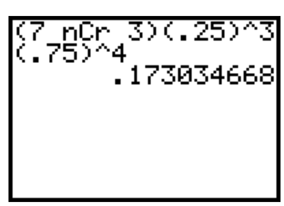
Section 7.2 ▪ The Binomial Distribution

Binomial Probabilities 2nd [DISTR] DISTR 0:binompdf(

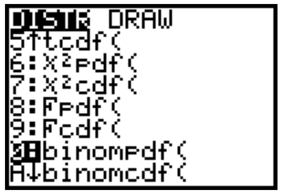
The TI-83 and TI-83 Plus can calculate binomial probabilities in two ways:

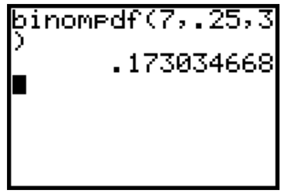
1. Use the formula $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$ and enter each factor individually. To get the binomial coefficient $\binom{n}{k}$, first enter the number to choose from, then press MATH **PRB 3:nCr**, and then enter the number to choose.



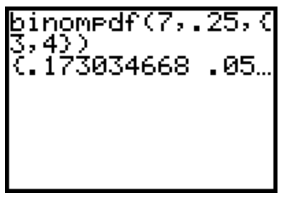


2. Use the binomial probability density function. You find the **binompdf(** command by pressing [2nd] **[DISTR] DISTR 0:binompdf(**. The syntax is **binompdf(***n*, *p*, *k*). This command calculates the binomial probability for *k* successes out of *n* trials when the probability of success on any one trial is *p*.

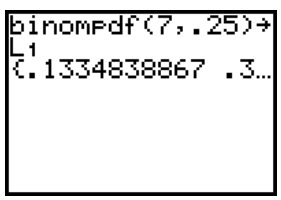


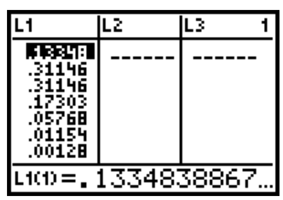


The number of successes can also be entered as a list of numbers set in braces.



If *k* is omitted, the entire binomial probability distribution is displayed. You can scroll through the list of probabilities using the left and right arrow keys. You could also store the probabilities in a list. There are three ways to do this: press [STO→] and the name of a list after the **binompdf** command; press [STO→] and the name of a list immediately after calculating the **binompdf** (the Home screen will automatically begin the entry line with **Ans**); or define the list with the **binompdf** command in the List Editor.



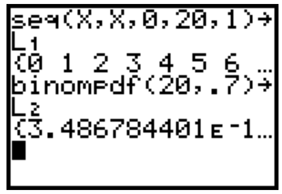


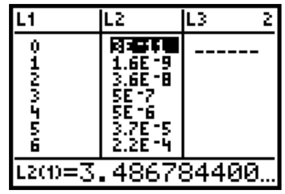
Note that a binomial distribution is a discrete distribution, as opposed to a normal distribution, which is continuous. Therefore, **binompdf(** cannot be used in the Y= screen in the same way that **normalpdf(** is. For example, **Y1=binompdf(7,.25,X)** does not result in a graph. See the next section for instructions about graphing a binomial distribution.

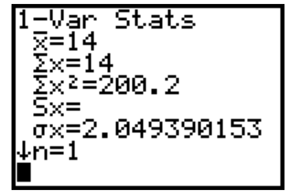
The Shape, Center, and Spread of a Binomial Distribution

If you enter all values of *X* into list **L1** and the binomial probabilities of *k* successes out of *n* trials into list **L2**, you can use the **1-Var Stats** command to calculate the expected value and standard deviation of the binomial distribution. This example shows a binomial distribution with $n = 20$ and $p = .7$. The section "The Law of

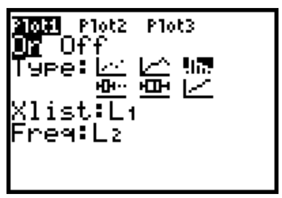
Large Numbers" in Chapter 6 of this Calculator Guide showed you how to use the sequence command to enter a sequence of integers.

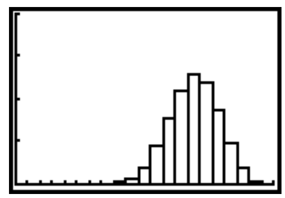






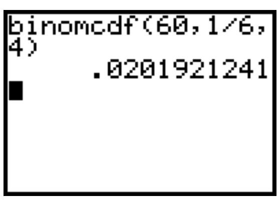
Once the binomial distribution is entered into two lists, you can display the distribution in a histogram. This histogram is the same as the bottom-right plot in Display 7.7 on page 396 of the *Statistics in Action* student text. (*Note:* In order to duplicate the histogram as it appears in the *Statistics in Action* student text, be sure to set the window [0, 21, 1, 0, 0.3, 0.075], especially setting **Xscl=1**.)





[0, 21, 1, 0, 0.3, 0.075]

Binomial Cumulative Distribution   2nd [DISTR] DISTR A:binomcdf(



To calculate the cumulative probability of a binomial distribution, use the binomial cumulative distribution function. You find this command by pressing 2nd **[DISTR] DISTR A: binomcdf(**, and the syntax is **binomcdf(***n, p, k***)**. For example, for Exercise 9 on page 398 of the *Statistics in Action* student text, **binomcdf(60,1/6,4)** gives the probability of fewer than five doubles (or the cumulative probability of 0, 1, 2, 3, or 4 doubles) when you roll a pair of dice 60 times.

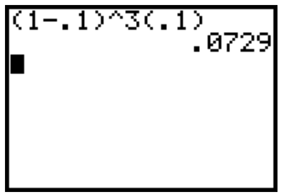
To find the probability of five or more doubles, subtract the result from 1.

Section 7.3 ▪ The Geometric Distribution

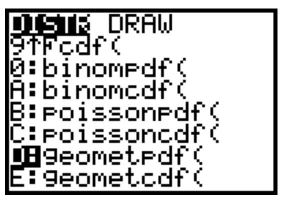
The Geometric Distribution 2nd [DISTR] DISTR D:geometpdf(

Similar to binomial probabilities, geometric probabilities can be calculated in two ways:

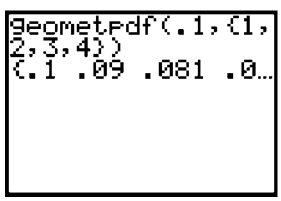
1. Use the formula $P(X = k) = (1 - p)^{k-1}p$.



2. Use the geometric probability density function. You find the **geometpdf(** command by pressing 2nd **[DISTR] DISTR D:geometpdf(**. The syntax is **geometpdf(***p, k*). This command calculates the probability that the first success occurs on the *k*th trial, where *p* is the probability of each success.

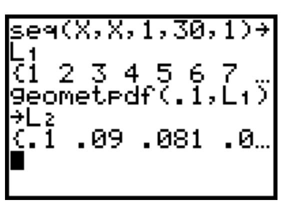


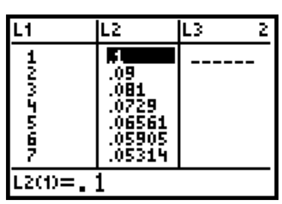
The parameter *k* can also be entered as a list of numbers set in braces.



Because any geometric distribution has an infinite number of values, *k* cannot be omitted.

The probabilities for a finite sequence of values of *k* can be calculated, stored in the List Editor, and graphed. Here is the geometric distribution for $p = .1$, as shown in the first plot in Display 7.8 on page 401 of the *Statistics in Action* student text.





[0, 31, 1, 0, 0.1, 0.02]

Geometric Cumulative Distribution 2nd [DISTR] DISTR E:geometcdf(

To calculate the cumulative probability of a geometric distribution, use the geometric cumulative distribution function. You find this command by pressing 2nd [DISTR] **DISTR E:geometcdf(**. For example, for Practice 11b on page 404 of the *Statistics in Action* student text, **geometcdf(.91,3)** gives the probability of the first nondefective engine before the fourth trial (or the probability of success on the first, second, or third trial).

Expected Value and Standard Deviation

Although a geometric distribution has an infinite number of values, you can only store a finite sequence of values of *k* in the List Editor. So, if you use the **1-Var Stats** command to approximate the expected value and standard deviation, the resulting values will be too small. To get exact values, use the formulas given on page 405 of the *Statistics in Action* student text.