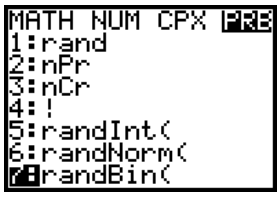
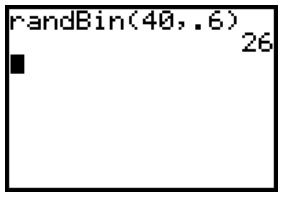
CHAPTER 8 INFERENCE FOR PROPORTIONS

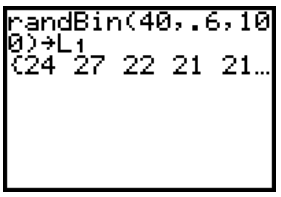
Section 8.1 ▪ Estimating a Proportion with Confidence

Discussion: Reasonably Likely Outcomes

Several times in this section and chapter, you are asked to simulate the reasonably likely outcomes for a binomial experiment. For example, Discussion 2a on page 415 of the *Statistics in Action* student text asks you to simulate the number of successes in a sample of size 40 from a population with 60% successes. The random binomial generator, **randBin(**, allows you to do this. You find the command by pressing MATH **PRB 7:randBin(**. The command's syntax is **randBin(***number of trials, probability of success, number of simulations***)**; if *number of simulations* is omitted, the default is 1. For example, **randBin(40,.6)** returns a single number that represents the number of successes in a sample of 40. The command **randBin(40,.6,100)** returns the number of successes for 100 simulations, which can be stored in a list.



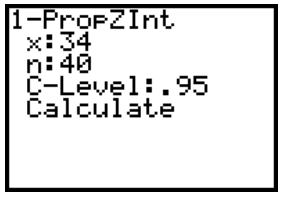


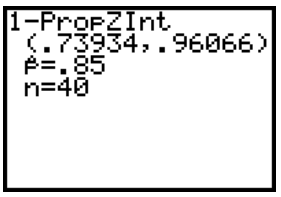


The Meaning of a Confidence Interval STAT TESTS

A:1-PropZInt

The TI-83 and TI-83 Plus calculate the confidence interval for a proportion with the command **1-PropZInt**. You find this command by pressing STAT **TESTS A:1-PropZInt**. Enter the number of successes in the sample, **x**, the sample size, **n**, and the level of confidence, **C-Level**. Arrow down and select **Calculate** to get the confidence interval and sample proportion. These screens show the calculations for discussion question D5 on page 419 of the *Statistics in Action* student text. The results show that then 95% confidence interval for the percentage of toddlers who know Barney's color is approximately 74% to 96%.



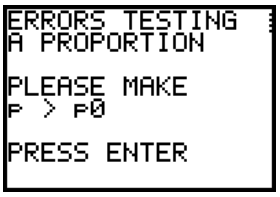


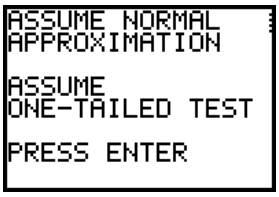
Section 8.2 ▪ Testing a Proportion

Types of Errors

The program ERRORS can help you understand Type II errors and power. The program graphically shows the effects that changing parameters has on the probability of a Type II error and the power of the test.

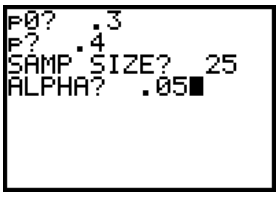
a. Run the program: press PRGM EXEC, choose ERRORS, and press ENTER. Read the introductory screens, which emphasize that the program uses a normal approximation and a one-tailed test where $p > p_0$. Press ENTER after each screen.



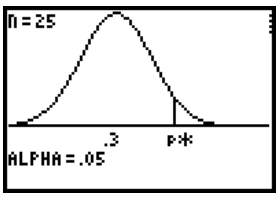


b. At the prompts, enter the hypothesized proportion you are testing, P0, the true value of the population proportion, P, the sample size, and the significance level, α(ALPHA). The hypothesized proportion, P0, can be any value less than the true proportion. Press ENTER after each value.

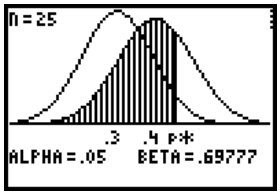
For example, suppose you plan to take a sample of size 25 to test that $p = .3$. Although you don't know it, the true proportion is $p = .4$. The null hypothesis that $p = .3$ is false.



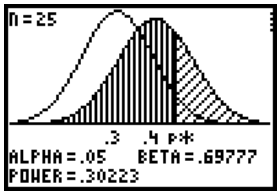
c. The program first graphs a normal approximation of the sampling distribution for the hypothesized proportion p_0. The critical value p^* is labeled. The null hypothesis will (correctly) be rejected if $\hat{p}$ falls above this critical value. The null hypothesis will not (incorrectly) be rejected if $\hat{p}$ falls below p^*.



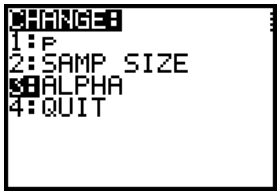
d. Pressing ENTER graphs a normal approximation of the sampling distribution for the true population proportion. The tail is highlighted below the critical value p^*, which represents the probability of *not* rejecting this false null hypothesis. The area of this tail, called BETA, or β, represents the probability of a Type II error.

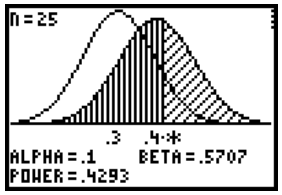


e. Pressing ENTER again shades the second curve above the critical value. This area, $1 - \beta$, represents the power of the test. It represents the probability of rejecting this false null hypothesis.



f. Pressing ENTER gives you the option to change the true population proportion, the sample size, or the significance level. Changing the parameters shows the effects on the probability of a Type II error. For example, if you increase the significance level, the probability of a Type II error decreases. But if you were correct in the first place that $p = .3$, then you have increased the probability of a Type I error.





In practice, it is usually impossible to compute the probability of a Type II error and the power of the test because you have to know the value of p. In that way, ERRORS only allows you to explore the effects of changing parameters.

```
PROGRAM:ERRORS
ClrHome
PlotsOff
FnOff
AxesOn
Disp "ERRORS TESTING"
Disp "A PROPORTION"
Disp ""
Disp "PLEASE MAKE"
Disp "p > p0"
Disp ""
Disp "PRESS ENTER"
Pause
ClrHome
Disp "ASSUME NORMAL"
Disp "APPROXIMATION"
Disp ""
Disp "ASSUME"
Disp "ONE-TAILED TEST"
Disp ""
```

(continued)

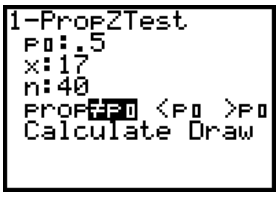
(PROGRAM: ERRORS continued)

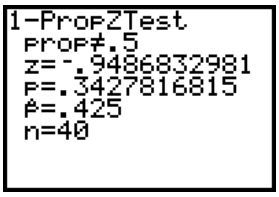
```
Disp "PRESS ENTER"
Pause
ClrHome
Input "p0?  ",P
Input "p?  ",Q
If P≥Q
Then
Disp "PLEASE MAKE"
Disp "p > p0"
Disp "p0  ",P
Input "p?  ",Q
End
Input "SAMP SIZE?  ",N
Input "ALPHA?  ",A
Lbl D
invNorm(1-A,P,√(P(1-P)/N)→Z
"normalpdf(X,P,
      √(P(1-P)/N)"→Y1
"normalpdf(X,Q,
      √(Q(1-Q)/N)"→Y2
FnOff
P-3√(P(1-P)/N)→Xmin
Q+3√(Q(1-Q)/N)→Xmax
0→Xscl
max(Y1(P),Y2(Q))+.02→Ymax
-Ymax/2→Ymin
0→Yscl
ClrDraw
DrawF Y1
Text(0,0,"N= ",N)
Text(43,round(91(P-Xmin)/
      (Xmax-Xmin),0)-2,P)
Text(43,round((91(Z-Xmin)/
      (Xmax-Xmin)),0),"p*")
Text(50,0,"ALPHA = ",A)
Line(Z,0,Z,Y1(Z))
Pause
Text(43,round(91(Q-Xmin)/
      (Xmax-Xmin),0)-2,Q)
Shade(0,Y2,Xmin,Z,1,2)
Line(Z,0,Z,Y2(Z))
normalcdf(-1E99,Z,Q,
      √(Q(1-Q)/N)→B
round(B,5)→B
Text(50,47,"BETA = ",B)
Pause
Shade(0,Y2,Z,Xmax,4,4)
Text(57,0,"POWER = ",1-B)
Pause
ClrHome
Menu("CHANGE:","p",A,"SAMP
      SIZE",B,"ALPHA",C,
      "QUIT",E)
Lbl A
Input "NEW p?  ",Q
If P≥Q
Then
Disp "PLEASE MAKE"
Disp "p > p0"
Disp "p0  ",P
Input "NEW p?  ",Q
End
Goto D
Lbl B
Input "NEW SAMP SIZE?  ",N
Goto D
Lbl C
Input "NEW ALPHA?  ",A
Goto D
Lbl E
ClrHome
```

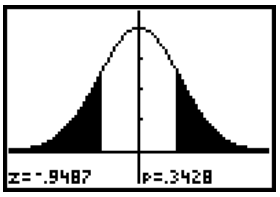
P-Values [STAT] TESTS 5:1-PropZTest

The TI-83 and TI-83 Plus conduct a significance test for a proportion and calculate the *P*-value with the command 1-PropZTest. You find the command by pressing [STAT] TESTS 5:1-PropZTest. At the prompts, enter the proportion under the null hypothesis, p_0, the number of successes in the sample, x, and the sample size, n. (*Note:* If you are given $\hat{p}$ instead of x, calculate x by multiplying $\hat{p} \cdot n$ and rounding to the nearest integer.) Select the type of test: two-tailed ($\neq p_0$) or one-tailed ($< p_0$ or $> p_0$). Arrow down and select Calculate to get the value of the test statistic, z, the *P*-value, p, the sample proportion, $\hat{p}$, and the sample size.

Select **Draw** to get a shaded distribution. The screens below show the process for Jenny and Maya as shown on pages 445–446 of the *Statistics in Action* student text.





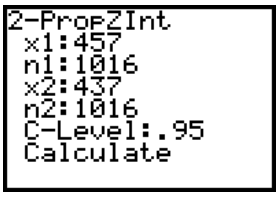


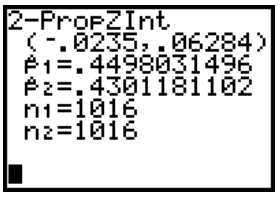
As outlined on page 439 of the *Statistics in Action* student text, statisticians have chosen clear and definitive guidelines for the presentation of a test of significance. It is critical that you understand the appropriate use of calculator technology within this presentation. You should use your calculator only after writing the name of the test, checking the criteria for its use, stating the hypotheses, and stating the test statistic. At that point, it is appropriate to have the calculator perform the test and calculate the *P*-value. Complete the significance test by stating whether or not you reject H_0 and writing a conclusion in context.

Section 8.3 ▪ A Confidence Interval for the Difference of Two Proportions

The Formula for the Confidence Interval STAT TESTS B:2-PropZInt

The TI-83 and TI-83 Plus calculate the confidence interval for the difference of two proportions with the command **2-PropZInt**. You find this command by pressing STAT **TESTS B:2-PropZInt**. At the prompts, enter the number of successes in each sample, **x1** and **x2**, both sample sizes, **n1** and **n2**, and the level of confidence, **C-Level**. Arrow down and select **Calculate** to get the confidence interval, both sample proportions, and both sample sizes. Note that **2-PropZInt** does not ask for the sample proportions. So, if you are given the proportions instead of the number of successes, multiply the proportions by the sample size and round to the nearest whole number. For example, with the dog-ownership example on pages 456–457 of the *Statistics in Action* student text, use **x1** = 457 and **x2** = 437.





Section 8.4 ▪ A Significance Test for the Difference of Two Proportions

The Theory of a Significance Test for the Difference of Two Proportions [STAT] TESTS 6:2-PropZTest

The TI-83 and TI-83 Plus conduct a significance test for the difference of two proportions with the command **2-PropZTest**. You find this command by pressing [STAT] **TESTS 6:2-PropZTest**. The command behaves similar to the **1-PropZTest** explained in the section "*P*-Values." These screens show the calculations for the AZT example on pages 466–467 of the *Statistics in Action* student text. Note that the input requires the numbers of successes instead of proportions and that the output includes the test statistic, the *P*-value, the sample proportions, and the pooled estimate, $\hat{p}$.